

Changing the subject of the formula

What do we mean by changing the subject of the formula ?

Consider the formula for the circumference of a circle.

$$C = \pi d$$

If we are given the diameter then we can use this formula to calculate the circumference.

However, if we are given the circumference, and need to calculate the diameter, then it would make more sense to have a formula of the form:

$$d = \dots\dots$$

We will look at how we can change around formulae, to make using them easier.

Terminology

In the formula

$C = \pi d$

Subject: C Rule: multiply π by *diameter*

The variable on the left, is known as the **subject**: What you are trying to find.

The formula on the right, is the **rule**, that tells you how to calculate the subject.

So, if you want to have a formula or rule that lets you calculate d , you need to make d , the **subject** of the formula.

This is changing the subject of the formula from C to d .

So clearly in the case above where

$$C = \pi d$$

We get C by multiplying π by the *diameter*

To calculate d , we need to divide the Circumference C by π

So

$$d = \frac{C}{\pi}$$

and now we have d as the subject of the formula.

Method:

A formula is simply an equation, that you cannot solve, until you replace the letters with their values (numbers). It is known as a **literal** equation.

To change the subject, apply the same rules as we have applied to normal equations.

1. Add the same variable to both sides.
2. Subtract the same variable from both sides.
3. Multiply both sides by the same variable.
4. Divide both sides by the same variable.
5. Square both sides
6. Square root both sides.

In Practice

There are a few things we should note:

1. Get rid of fractions – multiply throughout by the denominator.
2. Break brackets – if it helps.

Examples:

Make the **letter** in **brackets** the subject of the formula

$$x + p = q \quad [x]$$

(subtract p from both sides)

$$x = q - p$$

$$y - r = s \quad [y]$$

(add r to both sides)

$$y = s + r$$

$$P = RS \quad [R]$$

(divide both sides by R)

$$\frac{P}{R} = S$$

$$\frac{A}{B} = L \quad [A]$$

(multiply both sides by B)

$$A = LB$$

$$2w + 3 = y \quad [w]$$

(subtract 3 from both sides)

$$2w = y - 3$$

(divide both sides by 2)

$$w = \frac{y - 3}{2}$$

$$P = \frac{1}{3}Q \quad [Q]$$

(multiply both sides by 3— get rid of fraction)

$$3P = Q$$

$$T = \frac{2}{5}k \quad [k]$$

(multiply both sides by 5— get rid of fraction)

$$5T = 2k$$

(divide both sides by 2)

$$\frac{5T}{2} = k \quad \text{Note that: } \frac{5T}{2} \text{ is the same as } \frac{5}{2}T$$

$$A = \pi r^2 \quad [r]$$

(divide both sides by π)

$$\frac{A}{\pi} = r^2$$

(square root both sides)

$$\sqrt{\frac{A}{\pi}} = r$$

$$L = \frac{1}{2}(h - t) \quad [h]$$

(multiply both sides by 2)

$$2L = h - t$$

(add t to both sides)

$$2L + t = h$$

$$P = 4 + \frac{5}{w} \quad [w]$$

(subtract 4 from both sides)

$$P - 4 = \frac{5}{w}$$

(multiply both sides by w)

$$w(P - 4) = 5$$

(divide both sides by $(P - 4)$)

$$w = \frac{5}{P - 4}$$

$$d = \frac{k - m}{t} \quad [k]$$

(multiply both sides by t)

$$td = k - m$$

(add m to both sides)

$$td + m = k$$

Imagine a formula to be like an onion, to re-arrange it, you need to remove each layer away from the variable you want to be left as the subject.

The order in which you do this is important.

Generally, start at the **outside** of the formula and work your way in.

Look at

$$d = \frac{k - m}{t}$$

Here, the whole right hand side is **divided by t**

To peel this away – undo it, **multiply by t**

We are then left with

$$td = k - m$$

To peel away the m – undo it, **add m to both sides**

Onion

$$d = \frac{k - m}{t}$$

Some examples to try:

1. $y = x + 5$ [x]

2. $y = 5 - x$ [x]

3. $D = ST$ [S]

4. $P = IRT$ [I]

5. $\frac{P}{V} = k$ [P]

6. $px = q + r$ [x]

7. $V = Ah$ [h]

8. $v = u + at$ [t]

9. $V = I^2R$ [I]

10. $P = \frac{T}{R}$ [R]

11. $D = \frac{3}{4}G + T$ [G]

12. $E = mc^2$ [c]

13. $s = \frac{300}{d^2}$ [d]

14. $F = \frac{GMm}{r^2}$ [m]

15. $F = \frac{GMm}{r^2}$ [r]

Solutions are on the next page

Solutions

$$1. \quad y = x + 5 \quad [x] \quad x = y - 5$$

$$2. \quad y = 5 - x \quad [x] \quad x = 5 - y$$

$$3. \quad D = ST \quad [S] \quad S = \frac{D}{T}$$

$$4. \quad P = IRT \quad [I] \quad I = \frac{P}{RT}$$

$$5. \quad \frac{P}{V} = k \quad [P] \quad P = kV$$

$$6. \quad px = q + r \quad [x] \quad x = \frac{q+r}{p}$$

$$7. \quad V = Ah \quad [h] \quad h = \frac{V}{A}$$

$$8. \quad v = u + at \quad [t] \quad t = \frac{v-u}{a}$$

$$9. \quad V = I^2R \quad [I] \quad I = \sqrt{\frac{V}{R}}$$

$$10. \quad P = \frac{T}{R} \quad [R] \quad R = \frac{T}{P}$$

$$11. \quad D = \frac{3}{4}G + T \quad [G] \quad G = \frac{4(D-T)}{3}$$

$$12. \quad E = mc^2 \quad [c] \quad c = \sqrt{\frac{E}{m}}$$

$$13. \quad s = \frac{300}{d^2} \quad [d] \quad d = \sqrt{\frac{300}{s}}$$

$$14. \quad F = \frac{GMm}{r^2} \quad [m] \quad m = \frac{Fr^2}{GM}$$

$$15. \quad F = \frac{GMm}{r^2} \quad [r] \quad r = \sqrt{\frac{GMm}{F}}$$